The Connection Between Limits and Asymptotes

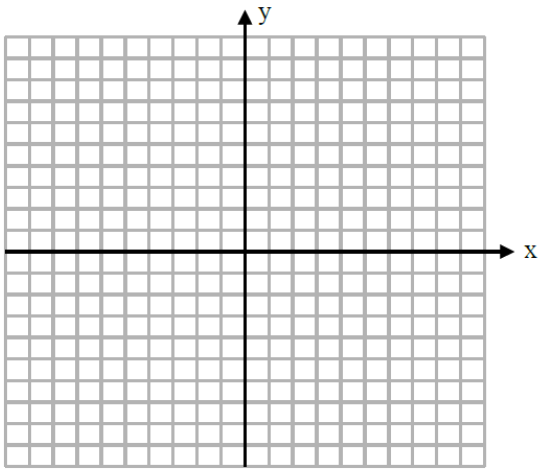
Part I: For each question,

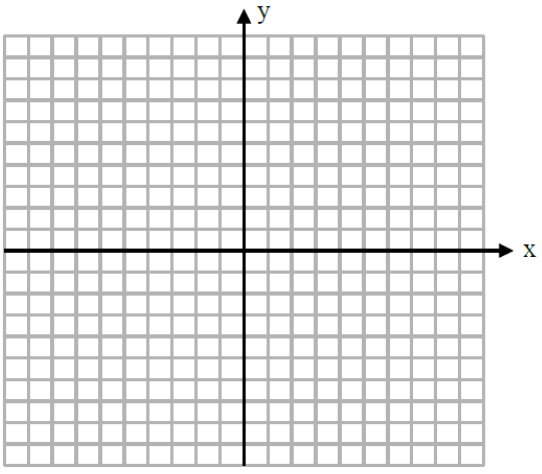
- Decide whether the given information allows you to identify any horizontal or vertical asymptotes for the graph of $y = f(x)$.
- If there is sufficient information, state the equations of any asymptotes.
- Draw a possible graph of $y = f(x)$.

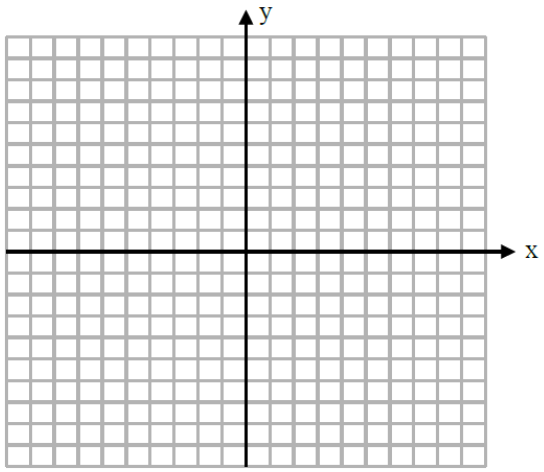
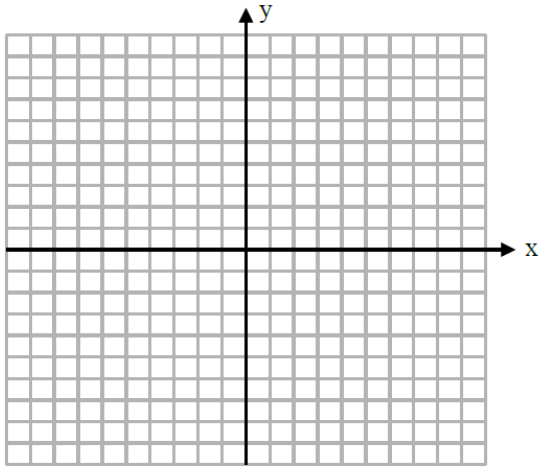
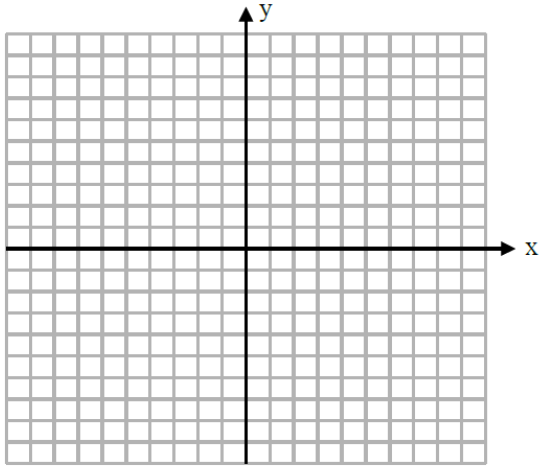
End behavior ($\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$):	Vertical asymptote (infinite discontinuity):
Order of Dominance:	

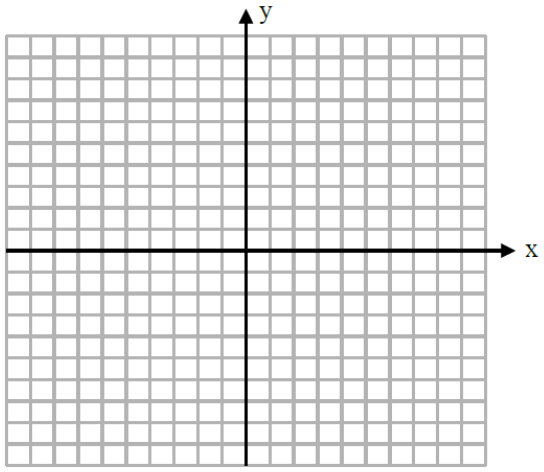
Example: $\lim_{x\to 3^-} f(x) = +\infty$, $\lim_{x\to 3^+} f(x) = -\infty$

- Vertical asymptote(s)?
  ☐ No ☐ Yes, equation(s):_____
- Horizontal asymptote(s)?
  ☐ No ☐ Yes, equation(s):_____



1	$\lim_{x\to 1} f(x) = \infty$, $f(1) = 4$ • Vertical asymptote(s)? ☐ No ☐ Yes, equation(s):_____ • Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s):_____	

2	$\lim_{x\to+\infty} f(x)=2$, $\lim_{x\to-\infty} f(x)=4$ • Vertical asymptote(s)? ☐ No ☐ Yes, equation(s):_____ • Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s):_____	
3	$\lim_{x\to-1^-} f(x)=4$, $\lim_{x\to-1^+} f(x)=-\infty$ • Vertical asymptote(s)? ☐ No ☐ Yes, equation(s):_____ • Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s):_____	
4	$\lim_{x\to4^-} f(x)=2$, $\lim_{x\to4^+} f(x)=3$ • Vertical asymptote(s)? ☐ No ☐ Yes, equation(s):_____ • Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s):_____	

5	$\lim_{x\to 4^-} f(x) = -\infty,\ \lim_{x\to 4^+} f(x) = +\infty$ $\lim_{x\to -\infty} f(x) = 5,\ \lim_{x\to +\infty} f(x) = -3$ • Vertical asymptote(s)? ☐ No ☐ Yes, equation(s):_____ • Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s):_____	

Part II: Evaluate each limit. State any horizontal or vertical asymptotes.

Ex. 1 $\lim_{x\to 0^+} \frac{-1}{6x}$	**Ex. 2** $\lim_{x\to 3^+} \frac{5}{x-3}$
Ex. 3 $\lim_{x\to 5^-} \frac{x+3}{x-5}$	**Ex. 4** $\lim_{x\to\infty} \frac{11x^2+4x-7}{x^3-\ln x} - 6$
Ex. 5 $\lim_{x\to\infty} \frac{10x}{\sin 14x}$	**Ex. 6** $\lim_{x\to -\infty} \frac{5x^3}{e^x - 4x^3}$